Pseudorandom generator (PRG)

Pseudorandom Generator (PRG)

- Given a seed, it outputs a sequence of random bits

  PRG(seed) -> random bits

- It can output arbitrarily many random bits

PRG security

- Can PRG(K) be truly random?

No. Consider key length |K|=k. Have 2^k possible initial states of PRG. Deterministic from then on. There are more random states.

- A secure PRG suffices to “look” random (“pseudo”) to an attacker (no attacker can distinguish it from a random sequence)

Example of PRG: using block cipher in CTR mode

If you want m random bits, and a block cipher with E_k has n bits, apply the block cipher m/n times and concatenate the result:

PRG(K | IV) = E_k(IV|1) | E_k(IV| 2) | E_k(IV|3) … E_k(IV| ceil(m/n)), where | is concatenation

Application of PRG: Stream ciphers

- Another way to construct encryption schemes
- Similar in spirit to one-time pad: it XORs the plaintext with some random bits
- But random bits are not the key (as in one-time pad) but are output of a pseudorandom generator PRG

Application of PRG: Stream cipher

Enc(K, M):

- Choose a random value IV
- C = PRG(K | IV) XOR M
- Output (IV, C)

Q: How decrypt?

A: Compute PRG(K | IV) and XOR with ciphertext C

Q: What is advantage over OTP?

A: Can encrypt any message length because PRG can produce any number of random bits, and multiple times because IV is chosen at random in Enc

Discrete Logarithm Problem (DLP)

$f(X) = \underbrace{g^X \bmod p}_{y}$ where p is a large prime (2048 bits long)

g is a random value in $[2, p-1]$

Assumption: f_{DLP} is OWF

Easy to compute:

Say X is 2048-bit large number.

$\leq 2^{2048}$ $\quad 2^{128}$

repeated squaring

2^{32}

$2^{16} \cdot 2^{16}$

1 mult

$2^{16} \cdot 2 \cdot 2 \cdot 2 \cdots$

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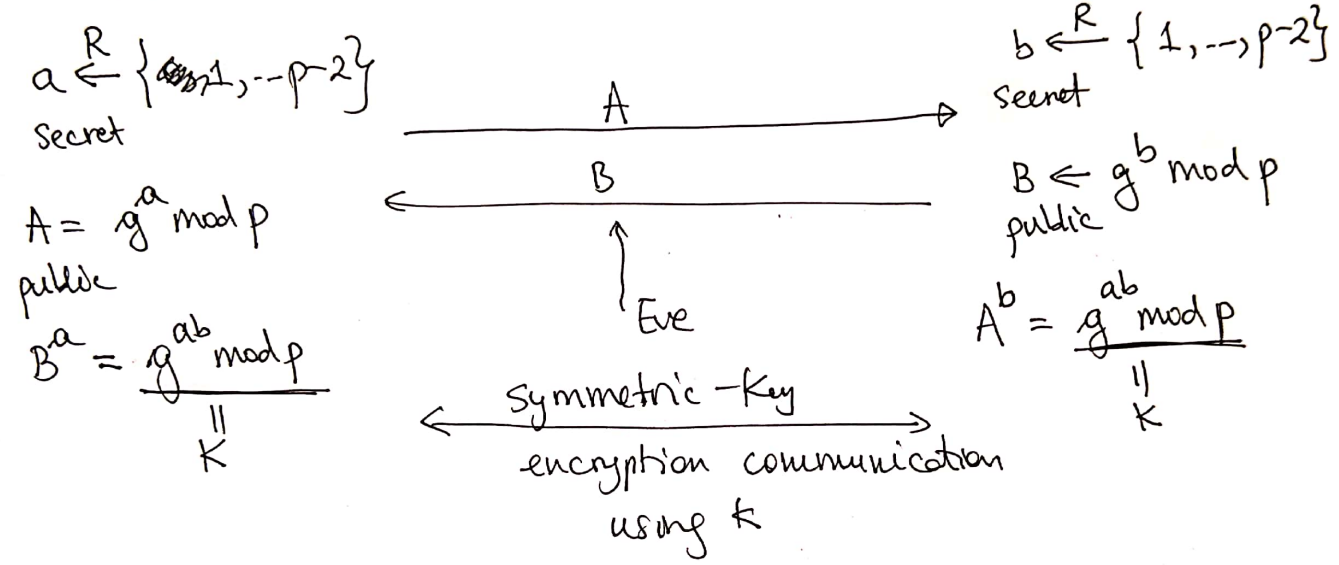
Diffie-Hellman Key Exchange (1976)

(Turing award)

large prime p, $1 < g < p-1$ — public



Eve sees: $A = g^a \bmod p \Rightarrow$ cannot compute a

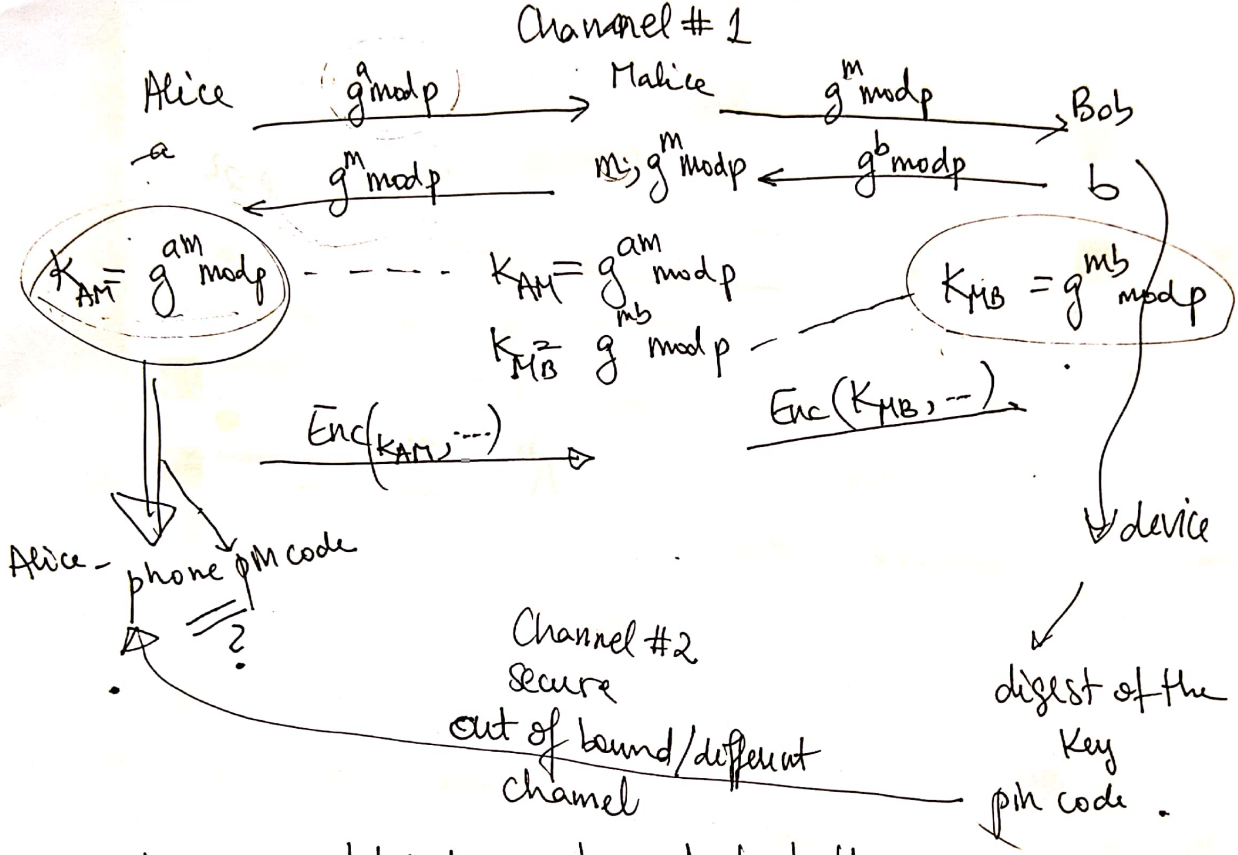
$B = g^b \bmod p \Rightarrow$ cannot compute b

} cannot compute g^{ab}

Assumption: you cannot break DLP (DLP is OWF) ← necessary

Adv cannot compute $g^{ab} \bmod p$ from $g, g^a, g^b \bmod p$

Man in the middle attack (MITM)



Assumes Adv does not control both channels